Useful Statistical Rules

Note: For this review sheet, assume that a and b are constants and X, Y, Z are random variables.

1. Rules for working with summation notation

To conveniently write out the sum of values x_i, from x_1 to x_n, we use the following notation:

$$\sum_{i=1}^{n} x_i \equiv x_1 + x_2 + x_3 + \cdots + x_n$$

The following rules apply for working with summation notation:

$$\sum_{i=1}^{n} ax_i = a\sum_{i=1}^{n} x_i$$

$$\sum_{i=1}^{n}(ax_i + by_i) = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} y_i$$

$$\sum_{i=1}^{n}(x_i + a) = na + \sum_{i=1}^{n} x_i$$

$$\sum_{i=1}^{n} a = na$$

Beware:

$$\sum_{i=1}^{n}(x_i y_i) \neq \Big(\sum_{i=1}^{n} x_i\Big)\Big(\sum_{i=1}^{n} y_i\Big)$$

$$\sum_{i=1}^{n} x_i^2 \neq \Big(\sum_{i=1}^{n} x_i\Big)^2$$

$$\sum_{i=1}^{n}\Big(\frac{x_i}{y_i}\Big) \neq \frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} y_i}$$

2. Expected values

The expected value of *discrete* random variable X is defined as:

$$\mathbb{E}[X] \equiv \mu_X = \sum_{i=1}^{\infty} x_i p_i \qquad \text{(Definition of expected value for a discrete R.V.)}$$

The expected value of *continuous* random variable X is defined as:

$$\mathbb{E}[X] \equiv \mu_X = \int_{-\infty}^{\infty} x f(x) dx \qquad \text{(Definition of expected value for a continuous R.V.)}$$

The following useful rules follow from the above definitions of expected values and rules for working with summation notation and are known together as "linearity of expectations":

$$\begin{aligned} \mathbb{E}[a + X] &= a + \mathbb{E}[X] \\ \mathbb{E}[aX] &= a\mathbb{E}[X] \\ \mathbb{E}[aX + bY] &= a\mathbb{E}[X] + b\mathbb{E}[Y] \qquad \text{(Linearity of expectations)} \end{aligned}$$

3. Variance rules

The variance of random variable X is defined as:

$$Var(X) \equiv \sigma_X^2 = \mathbb{E}[(X - \mathbb{E}(X))^2] \qquad \text{(Definition of variance)}$$

The definition of variance can be rewritten in this particularly useful form:

$$Var(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 \qquad \text{(Definition of variance, alternate form)}$$

The following rules for working with variances derive from the definition of variance:

$$\begin{aligned} Var(a) &= 0 \\ Var(a + X) &= Var(X) \\ Var(aX) &= a^2 Var(X) \\ Var(aX + bY) &= a^2 Var(X) + b^2 Var(Y) + 2ab\, Cov(X, Y) \\ \mathbb{E}[X] = 0 &\iff Var(X) = \mathbb{E}[X^2] \end{aligned}$$

4. Covariance rules

The covariance of random variables X and Y is defined as:

$$Cov(X, Y) \equiv \sigma_{XY} = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])] \qquad \text{(Definition of covariance)}$$

Analogous to the alternative form of the definition of variance is the following useful way of rewriting the definition of covariance:

$$Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y) \qquad \text{(Definition of covariance, alternate form)}$$

The following rules for working with covariances derive from the definition of covariance:

$$\begin{aligned} Cov(a+X, b+Y) &= Cov(X,Y) \\ Cov(aX, bY) &= ab\, Cov(X,Y) \\ Cov(X+Y, Z) &= Cov(X,Z) + Cov(Y,Z) \\ Cov(X,X) &= Var(X) \\ Cov(X,a) &= 0 \end{aligned}$$

The correlation between random variables X and Y is defined as:

$$Corr(X,Y) \equiv \rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

5. Rules for working with the normal distribution

For a normally distributed random variable X with Mean μ and variance σ^2:

$$X \sim \mathcal{N}(\mu, \sigma^2) \implies \frac{X-\mu}{\sigma} \sim \mathcal{N}(0,1)$$

$$\begin{aligned} \mathbb{E}[X] &= \mu \\ Var(X) &= \sigma^2 \end{aligned}$$