Interpreting Marginal Rates of Substitution

1. Definition of MRS

Marginal Rate of Substitution *The rate at which you would be willing to trade one good (say x_1) for another good (x_2) while remaining indifferent to the trade. The MRS at a point can be interpreted as the slope of an indifference curve at that point.*

$$MRS = \left|\frac{-MU_{x_1}}{MU_{x_2}}\right| = \left| - \frac{\partial U/\partial x_1}{\partial U/\partial x_2}\right|$$

Proof. The total change in utility given some in x_1 and x_2 can be approximated as:

$$dU = \frac{\partial U}{\partial x_1}dx_1 + \frac{\partial U}{\partial x_1}dx_1$$

This says that the total change in utility is the change in x_1 times the amount utility changes with a change in x_1 while holding x_2 constant, plus the change in x_2 times the amount utility changes with a change in x_2 while holding x_1 constant. If we hold utility constant, then change in utility (dU) is zero, so:

$$0 = \frac{\partial U}{\partial x_1}dx_1 + \frac{\partial U}{\partial x_1}dx_1$$

Rearranging, we find that:

$$\frac{dx_2}{dx_1} = \frac{-\partial U/\partial x_1}{\partial U/\partial x_2} \tag{1}$$

This says that when we hold utility constant, say along an indifference curve, the slope of that curve is given by the MRS as defined above. Remember that slope is typically change in y over change in x, so if we plot x_2 on the y-axis and x_1 on the x-axis, we can easily interpret the left side of (1) as slope of an indifference curve. The right side of (1) is the definition of MRS, though we often work with the absolute value by convention. ■

Note: Don't get confused by the fact that MU_{x_1} is in the numerator of the MRS and MU_{x_2} is in the denominator of the MRS. This proof shows that you need to interpret the MRS as the increase in x_2 you would be willing to accept for a one unit reduction in consumption of x_1. Alternatively, you can interpret it as the amount of x_2 you would be willing to give up for a one unit increase in consumption of x_1.

2. Example

If Harry has utility function $U_H(x_a, x_b) = x_a x_b^2$, and he currently has 3 apples and 2 bananas, would he be willing to give up 1 apple in exchange for a banana?

Answer: Harry's $MU_a = x_b^2$ and his $MU_b = 2x_a x_b$, so his MRS is:

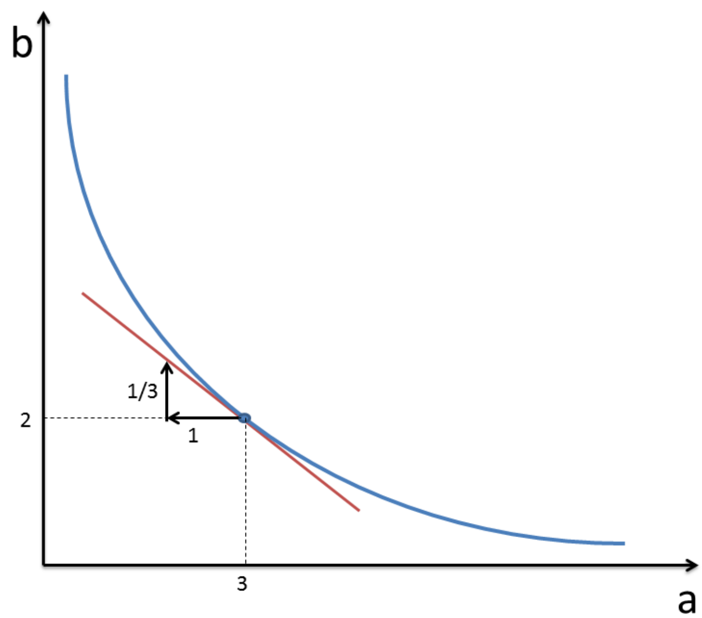
$$MRS = \frac{x_b^2}{2x_a x_b} = \frac{x_b}{2x_a}$$

At his current bundle, his MRS is evaluated as:

$$MRS(3,2) = \frac{2}{2(3)} = \frac{1}{3}$$

Graphically, this can be drawn:

Figure 1: Indifference curve over apples and bananas



As shown in the figure, at Harry's current endowment, his indifference curve has a slope equal to the MRS evaluated at that point (1/3). This says Harry would be indifferent to a loss of 1 apple and a gain of 1/3 of a banana. An equivalent way of interpreting this is that Harry would be willing to lose 3 apples in exchange for a gain of 1 banana.
Returning to the question, then, if Harry is willing to give up 3 apples in exchange for 1 apple, he is certainly willing to give up 1 apple in exchange for 1 banana.